Coherence absorption and condensation induced by thermalization of incoherent nonlinear fields

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Abstract – We show that a conservative system of incoherent nonlinear waves exhibits, as a rule, an irreversible process of coherence transfer, in which the incoherence of the system is absorbed by the small-amplitude field, thus allowing the high-amplitude field to evolve towards a highly condensed coherent state. This process of coherence absorption results from the natural thermalization of the fields to a thermodynamic equilibrium state. The theory reveals that, contrary to a classical gas system, a wave system does not satisfy an equipartition of energy among the particles. Such a distinctive feature is the key property underlying the existence of the coherence absorption process. The coherence absorption effect is shown to also occur in a system of quantum Bose gases.

The complex dynamics inherent to the vector nature of nonlinear wave phenomena has been the subject of a recent significant interest because of their experimental realizations in a variety of physical contexts. In the emerging key area of Bose–Einstein condensates, the simultaneous trapping of multicomponent gases was shown to exhibit unique properties not found in the pure condensate case \([1]\). Vector phenomena also play an important role in the framework of polarization optics due to the vectorial nature of the electromagnetic field. The richness of polarization nonlinear dynamics was shown to give rise to a multitude of novel behaviors inherent to the vector nature of nonlinear wave phenomena \([2]\).

In the present work we analyze the properties of coherence of coupled nonlinear wave systems. In this context, a remarkable progress has been recently accomplished since the first experimental demonstration of incoherent optical solitons in slowly response nonlinear materials \([3]\). The incoherent soliton consists of a phenomenon of self-trapping of spatially and temporally incoherent light in a biased photorefractive crystal \([2]\). Incoherent solitons and remarkable dynamical features inherent to incoherent nonlinear fields have also been recently reported in the framework of instantaneous nonlinearities \([4–9]\).

We consider the vector nonlinear Schrödinger (NLS) equation that provides a universal description of coupled nonlinear wave phenomena \([1,2,10,11]\). Our analysis reveals that a pair of incoherent waves exhibit, as a general rule, an irreversible process of coherence transfer, in which noise fluctuations are transferred from the high- to the small-amplitude field. The small-amplitude field then “absorbs” almost all the incoherence of the system, which allows the high-amplitude field to evolve towards a highly condensed coherent state. Let us stress that the phenomenon of coherence absorption constitutes a generic property of a coupled nonlinear wave system, in the sense that it results from the natural tendency of the isolated (Hamiltonian) system to approach a thermodynamic equilibrium state.

We analyze the thermalization process of the coupled wave system by making use of the kinetic wave theory \([12]\). The kinetic approach relies on a natural asymptotic closure induced by the dispersive properties of the waves, which leads to a kinetic description of the wave interaction that is formally based on irreversible kinetic equations. Such irreversible behavior is expressed by the $H$-theorem of entropy growth \([12]\), in analogy with the Boltzmann’s $H$-theorem relevant for gas kinetics \([13]\). Accordingly, the coherence absorption process finds its origin in the natural thermalization of the waves towards their equilibrium.

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Einstein condensates in optics, plasma, hydrodynamics or Bose-Einstein condensates provide a representative model of vector wave phenomena coherent to the incoherent that occurs from the high- to the small-amplitude field. This thermalization process appears counterintuitive because it is based on a transfer of noise fluctuations that occurs from the coherent to the incoherent component, i.e., a transfer of “heat from the cold to the hot” component. The noise in the system then results to be stored within the small-amplitude field, rather than being uniformly distributed among the wave packets, a feature which contrasts with the intuitive idea of energy equipartition. The theory reveals that, contrary to a classical gas system, a wave system does not exhibit an equipartition of energy among the particles, a distinctive feature which is responsible for the existence of the coherence absorption process. Moreover, the effect of coherence absorption is shown to also occur in a mixture of ideal quantum Bose gases. In this context, a dilute species “absorbs the noise” of the dense species, which in turn leads to the condensation of the dense species.

We consider a system of $M$ coupled NLS equations that provide a representative model of vector wave phenomena in optics, plasma, hydrodynamics or Bose-Einstein condensates [1]

$$i\partial_t \psi_j = -\rho_j \nabla^2_D \psi_j + \left( |\psi_j|^2 + \kappa \sum_{i \neq j} |\psi_i|^2 \right) \psi_j,$$

where $\nabla^2_D$ refers to the Laplacian operator in space dimension $D$. For convenience, we normalized the problem with respect to the characteristic nonlinear interaction time $\tau_0 = 1/(\gamma \epsilon_0^3)$ and length $\Lambda_0 = (\alpha_1 \tau_0)^{1/2}$, where $\gamma$ is the nonlinear coefficient, $\alpha_1$ the dispersion coefficients of $\psi_j$, and $\epsilon_0^3$ the average “intensity” of the field $\psi_1$. The variables can be recovered in real units through the transformations $t \to t\tau_0; r \to r\Lambda_0$ and $\psi_j \to \psi_j \tau_0$. With these units the dimensionless dispersion coefficients read $\rho_j = \sqrt{\alpha_1/\alpha_1}$. The last term in eqs. (1) describes a cross-interaction between a pair of distinct wave packets, $\psi_j$ and $\psi_{i \neq j}$, The dimensionless parameter $\kappa$ then denotes the ratio between the cross- and the self-interaction coefficients. In the following we restrict our analysis to $\rho_j > 0$ and $\kappa < 1$ so as to avoid modulational instabilities and segregation processes between the fields [1,2].

Equation (1) conserves the “amplitudes” $N_j = \int |\psi_j|^2 \, \, dr/L^D$ of each field $\psi_j$, $L^D$ being the system size. In the context of Bose-Einstein condensates, $N_j$ refers to the density of particles of the species $\psi_j$. We underline that, contrary to the usual consideration that deals with the dynamics of coherent fields $\psi_j$ [1,14], here we study the dynamics of incoherent (stochastic) matter waves, which provides a classical description of finite-temperature thermal Bose gases [15].

A physical insight into the process of coherence absorption may be obtained by means of the numerical integration of eqs. (1). A typical evolution of $M=2$ incoherent fields characterized by an amplitude ratio $N_2/N_1 = 0.2$ is illustrated in fig. 1. The random fields are assumed to be of zero mean and to obey a homogeneous statistics. Figure 1 remarkably shows that the spectrum of $\psi_1$ gets narrower to the detriment of a spectral broadening of $\psi_2$. The spectral width $\sigma_j$ of $\psi_j$ being inversely proportional to its correlation length $\lambda_j$ [13], fig. 1 reveals that the incoherence of $\psi_1$ is transferred to $\psi_2$. This allows $\psi_1$ to reach a highly coherent state, i.e. almost a plane wave in which the spectrum of $\psi_1$ is essentially concentrated in the mode $k = 0$. In the following we shall see that the states of the fields at $t = 200$ actually correspond to their equilibrium states. Accordingly, the process of coherence absorption illustrated in fig. 1 will appear as a natural consequence of the thermalization of the fields $\psi_j$ towards their thermodynamic equilibrium states.

The thermalization of a nonlinear wave may be considered as an irreversible process of diffusion in phase-space, which is due to the nonintegrable character of eqs. (1) [12]. It results that, despite the formal reversibility of the vector NLS eqs. (1), the fields $\psi_j$ exhibit an irreversible evolution to thermal equilibrium [5,7–9,15,16]. The essential properties of this irreversible relaxation process are described by the kinetic wave theory. Starting from the vector NLS eqs. (1), one can derive a set of irreversible kinetic equations describing the coupled evolutions of the averaged spectra of the fields $\bar{\psi}_j(k,t) = \langle |\psi_j(k,t)|^2 \rangle$ [12].

\[ \bar{\psi}_j(k,t) = \int \bar{\psi}_j(r,t) e^{-ik\cdot r} \, dr \]

being the Fourier transform of $\bar{\psi}_j$.
of $\psi_j(r,t)$ ($\langle \cdot \rangle$ denoting an average over the realizations):

$$\partial_t n_j(k_1,t) = \text{Coll}[n_j, n_j] + \kappa^2 \sum_{i \neq j} \text{Coll}[n_i, n_j].$$

(2)

The cross-collision term reads

$$\text{Coll}[n_i, n_j] = \int dk_2 \, dk_3 \, dk_4 \, N \, W_{i,j},$$

(3)

with

$$N = n_j(k_1) n_i(k_2) n_i(k_3) n_j(k_4) \left[ n_j^{-1}(k_1) + n_i^{-1}(k_2) - n_i^{-1}(k_3) - n_j^{-1}(k_4) \right],$$

(4)

where “$n_i(k_j)$” stands for $n_i(k_j,t)$. This collision term provides a kinetic description of the cross-interaction term in eqs. (1), i.e., the interaction between a pair of distinct wave packets $\psi_i$ and $\psi_j \neq i$. In simple words, the kinetic approach models the four-wave interaction as a collisional gas of quasi-particles satisfying the resonant conditions of energy and momentum conservation at each elementary collision. This is expressed by the presence of Dirac’s $\delta$-functions in the term $W_{i,j} = \delta(k_1 + k_2 - k_3 - k_4) \delta(\omega_i(k_1) + \omega_i(k_2) - \omega_i(k_3) - \omega_i(k_4))$, where $\omega_i(k) = \rho^2 k^2$ is the dispersion relation of $\psi_i$. In a similar way, the self-collision term $\text{Coll}[n_i, n_i]$ in eqs. (2) provides the kinetic description of the four-wave interaction of each individual field $\psi_j$.

In the following we shall focus on the equilibrium distributions of the vector kinetic eqs. (2). Equation (2) conserve the particle density (amplitude) $N_j = \int n_j(k,t) \, dk$ of each field $\psi_j$ and the total density of kinetic energy $E = \sum_i E_i$, $E_i = \int \omega_i(k) \, n_i(k,t) \, dk$.

The irreversible character of eqs. (2) is expressed by the $H$-theorem of entropy growth $dS \geq 0$, where the nonequilibrium entropy reads $S = \sum_i S_i$, $S_i = \int \log[n_i(k,t)] \, dk$.

The equilibrium state is determined from the postulate of maximum entropy [13]. The equilibrium spectra $n_j^{eq}(k)$ realizing the maximum of $S[n_j]$ subject to the constraints of conservation of $E$ and $N_j$, may be calculated by introducing the corresponding Lagrange’s multipliers, $1/T$ and $-\mu_j/T$. One readily obtains

$$n_j^{eq}(k) = \frac{T}{\rho_j k^2 - \mu_j},$$

(5)

where $T(\geq 0)$ denotes the temperature (representing the notion of thermal equilibrium among the $M$ fields $\psi_j$) and $\mu_j(\leq 0)$ the chemical potential of $\psi_j$ [5,7-9]. The $M+1$ unknown parameters $(T, \mu_j)$ can be calculated from the $M+1$ conserved quantities $(N_j, E)$. Accordingly, for a given nonequilibrium initial condition of $\psi_j$, characterized by the values of $(N_j, E)$, eqs. (5) determine the (asymptotic) equilibrium spectra of $\psi_j$ [5,7-9]. Important to note, the equilibrium distribution (5) consists of a Lorentzian spectrum, in which the chemical potential $\mu_j$ determines the correlation length of the field $\psi_j$ at equilibrium, $\lambda_j \propto 1/\sqrt{|\mu_j|}$. This correlation length being defined from the kinetic distribution (5), it is only relevant to the incoherent regime of interaction [17].

It proves convenient to write the equilibrium energy of $\psi_j$ in the following form: $E_j^{eq} = \int \omega_j(k) n_j^{eq}(k) \, dk = KT + \mu_j N_j$, where the parameter $K$ refers to the number of modes, $K = \pi k^2$ for $D = 2$, $K = \frac{4}{3} \pi k^2$ for $D = 3$, $k_e$ being a (ultraviolet) frequency cut-off [5,9,12]. Let us now make the important observation that, for a given system size $L^D$, the energy of a wave packet $E_j^{eq}$ must vanish as its amplitude $N_j$ tends to zero, i.e.,

$$\lim_{N_j \rightarrow 0} - \mu_j \rightarrow KT/N_j.$$  

(6)

Although the results presented in this letter are relevant to any space dimension $D$, let us analyze the meaning of the limit (6) in the two-dimensional case ($D = 2$) where an analytical expression for $\mu_j$ can be obtained in explicit form. Indeed, from the equilibrium spectrum (5) one readily obtains $N_j = \pi T \log(1 - \rho_j / \rho_j \mu_j) / \rho_j$, i.e., $\mu_j = (\rho_j K / \pi) / [1 - \exp(\rho_j N_j / \pi T)]$. This expression of $\mu_j$ may be substituted in the above expression of $E_j^{eq}$, which gives the “energy per particle”, $U_j^{eq} = E_j^{eq} / N_j$, of the field $\psi_j$ at equilibrium,

$$U_j^{eq} = \frac{KT}{N_j} - \frac{\rho_j K / \pi}{\exp(\rho_j N_j / \pi T) - 1}.$$  

(7)

We first analyze this expression in the particular case where the system is composed of a single field, $M = 1$. An inspection of eqs. (7) reveals that the temperature $T$ tends to zero as $N_j \rightarrow 0$ (holding $U_j^{eq}$ constant), i.e., $T \sim U_j^{eq} N_j / K$ for $U_j^{eq} \ll U_{j eq} \sim T_2 K / 2 \pi$ (see fig. 2a). According to the limit (6), this means that the chemical potential $\mu_1$, and the respective correlation length $\lambda_1$ of $\psi_1$, both tend to finite values as $N_1 \rightarrow 0$.

The problem is different for a vector system composed, e.g., of $M = 2$ fields characterized by $N_2 / N_1 \ll 1$. In this case, the high-amplitude field $\psi_1$ fixes a nonvanishing value of the temperature, $T > 0$. The limit (6) then reveals that the chemical potential $\mu_2$ tends to diverge, i.e., the small-amplitude field $\psi_2$ exhibits a significant spectral broadening as its amplitude $N_2$ decreases to zero.

This effect of coherence degradation of $\psi_2$ is due to a transfer of noise fluctuations from the high- to the small-amplitude field, i.e., from $\psi_1$ to $\psi_2$. To characterize such a process of coherence transfer, note that the evolution of the coherence length $\lambda_j(t)$ may be expressed in terms of the variance $\sigma_j(t) = 1 / \lambda_j(t)$ of the spectrum $n_j(k,t)$, i.e., $\sigma_j^2(t) = \int k^2 n_j(k,t) \, dk / \int n_j(k,t) \, dk = U_j(t) / \rho_j$. This expression shows that the energy per particle essentially determines the coherence length of the field $\psi_j$, $\lambda_j(t) = [\rho_j / U_j(t)]^{1/2}$.

The numerical simulations confirm the process of energy (i.e., coherence) transfer, as illustrated in fig. 2b, which shows the evolution of the energies per particle $U_j(t)$ for $M = 3$ interacting fields with $N_2 / N_1 = 0.15, N_3 / N_1 = 0.25$.

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In this example, the initial energies per particle were assumed identical, $U_1(t=0) = U_2(t=0) = U_3(t=0)$, i.e., the initial fields exhibit almost the same coherence. After a transient stage ($t \approx 200$), the three fields reach their equilibrium states. Accordingly, the energies per particle $U_j(t)$ evolve towards their corresponding equilibrium values $U_{j,eq}$ given by eqs. (7) (see fig. 2b). In this way the energy $U_j$ is transferred to the small-amplitude fields $\psi_{2,3}$, which store the incoherence of the system and thus allows $\psi_1$ to reach a highly coherent state (see fig. 1). This coherence absorption process solely results from the natural thermalization of the fields, as corroborated by the saturation of the process of entropy growth once the equilibrium state is reached, $dS \simeq 0$ (fig. 2c).

It is instructive to compare these results with those of a classical gas system. In this framework, the fundamental theorem of energy equipartition establishes that the energy is equally distributed among the particles, $E_{j,\text{gas}}^{eq}/N_j = U_{j,\text{gas}}^{eq} = Dk_B T/2$, where $k_B$ refers to Boltzmann’s constant [13]. Accordingly, a gas system composed of $M$ species thermalizes towards an equilibrium state characterized by the same energy per particle, $U_{j,\text{gas}}^{eq} = U_{j,\text{gas}}^{eq}$ (see the horizontal line in fig. 2b). As a consequence, the evolution of $U_j(t)$ illustrated in fig. 2b is forbidden in a gas system, because it corresponds to a transfer of “heat” (thermal energy) from the cold species (1) to the hot species (2), (3); The isolated system would spontaneously evolve away from equilibrium, which would violate the second law of thermodynamics [13].

There is obviously no contradiction with the second law, because the equilibrium properties of a wave system are fundamentally different from those of a gas system. Such a difference becomes apparent when the fields $\psi_j$ verify the condition $U_{j,eq}^{eq} \ll U_{j,eq}$ (see fig. 2b). As a result, the temperature $T$ is no longer fixed by the energy per particle $U_{j,eq}^{eq}$, but results to be proportional to the particle density $N_j$. For a given temperature $T$, the lower the density $N_j$, the higher the energy per particle $U_{j,eq}^{eq}$.

This merely explains the result of fig. 2b: the dilute species $\psi_{2,3}$ have to significantly increase their incoherence $U_{2,3}$ in order to reach the high temperature of the dense species $\psi_1$. We may thus state the following general property. Provided that the condition $U_{j,eq}^{eq} \ll U_{j,eq}$ is verified, a coupled wave system exhibits an irreversible evolution towards an equilibrium state, where each field $\psi_j$ shares the same amount of energy, $E_{j,eq}^{eq} = E_{j,eq}^{eq} = KT$, regardless of its initial coherence $|\langle \psi_j(t=0) \rangle|$, its amplitude $N_j$, and of the particular values of the dispersion parameters $\rho_j$, nonlinear coefficient $\kappa$, or space dimension $D$. This property has been remarkably well confirmed by the numerical simulations of eqs. (1), as illustrated in fig. 2d, in which the parameters have been chosen arbitrarily to underline the generality of the result.

Let us remark that, in spite of their differences, the equilibrium properties of a classical gas and of a wave system may be characterized by an equipartition of energy among the degrees of freedom: in a gas system, energy equipartition occurs among the number of particles $N_j$ ($E_{j,\text{gas}}^{eq}/N_j \propto T$), while in a wave system energy equipartition occurs among the number of modes $M$ ($E_{j,\text{gas}}^{eq}/M \propto T$). This aspect becomes apparent through the analysis of the spectra of the fields. When the fields verify the condition $U_{j,eq}^{eq} \ll U_{j,eq}$, the spectra are relatively narrow ($\sqrt{\rho_j} \ll k_c$) and the chemical potential may be neglected for large $k$. In this case the equilibrium distribution (5) exhibits a power law behaviour $n_j(k) \sim T/k^2$ for large $k$, as confirmed by the numerical simulations (see fig. 2c).

Accordingly, in frequency space the energy turns out to

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![Fig. 2](image-url)
be equally distributed among the modes of the system, \( \varepsilon_1(k) = \omega_1(k) n_{1,j}(k) \sim T \) [18]. Let us note, however, that the coherence absorption process gets more efficient when the energy per particle of the small-amplitude field approaches its maximum value \( U_{j,c} \). In this case the conditions \( U_j < U_{j,c} \) and \( \sqrt{\rho_j k^2} \ll k \) are no longer verified by the small-amplitude field, so that the coupled fields no longer share the same amount of energy \( E_j \) (i.e., \( E_1 \neq E_j \)) and the spectrum of the small-amplitude field departs from energy equipartition among the modes. The small-amplitude field then exhibits a broad Lorentzian spectrum (5) that prevents the establishment of energy equipartition, \( n_{1,j}(k) \neq T/k^2 \) (see fig. 2f). We finally remark that, as a result of the thermalization process, the small- and high-amplitude fields exhibit the same energy at large \( k \): \( T \sim \varepsilon_1(k) \sim \varepsilon_2(k) \), i.e., the tails of the spectra are of the same order, as confirmed by the numerical simulations (see figs. 1e, f).

The temperature of a field being proportional to its amplitude, \( T \sim U_{j}^{eq} N_{j}/K \), a small-amplitude field may be responsible for an effective “cooling” of a high-amplitude field. Let us now show that this cooling process may lead to a wave condensation of the high-amplitude field. This is illustrated in fig. 3a, in which the evolution of the condensate amplitude of \( \psi_1 \) is shown in the absence and in the presence of the dilute species \( \psi_{2,3} \).

The condensate amplitude, \( q_1(t) \equiv N_1^{eq}(t)/N_1 \), refers to the fraction of particles condensed in the fundamental mode \( k = 0 \). Here, the initial energies per particle were assumed identical, \( U_1(t = 0) = U_2(t = 0) = U_3(t = 0) \), i.e., the initial fields exhibit the same coherence. Figure 3 remarkably shows that the condensation of \( \psi_1 \) results to be solely induced by the interaction with the dilute species \( \psi_{2,3} \).

The condensation of a classical nonlinear wave governed by a NLS-like equation has witnessed a growing interest [5,6,9,15,16]. The condensation process is characterized by a critical energy \( U_{tr} \) below which particles spontaneously concentrate in the fundamental mode \( k = 0 \) [5]. Here, the condensation process is shown to result from the vector nature of the nonlinear wave system. To provide a simple analysis of this effect, we remark that the particle density \( (N_1^{eq}) \) condensed in \( k = 0 \) may be calculated by decomposing the sum as follows: \( N_j = N_j^{0} + T \sum_k 1/(\rho_j k^2 - \mu_j) \), where \( N_j^{0} = T/(-\mu_j) \) and \( \sum_k \) excludes the origin \( k = 0 \) [9]. Accordingly, the fraction of condensed particles of the field \( \psi_j \) reads

\[
q_1^{eq} = 1 - \frac{T}{N_j} \sum_k \frac{1}{\rho_j k^2 - \mu_j},
\]

where the \( M + 1 \) parameters \( (T, \mu_j) \) have been calculated by solving numerically the \( M + 1 \) equations: \( N_j^{0} = \sum_k T/(\rho_j k^2 - \mu_j) \), \( E = MKT + \sum_j \mu_j N_j \). Note that a more accurate analysis of vector wave condensation in the strong nonlinear regime would require Bogoliubov’s transformation (see the recent work [6]). Considering the relatively small values of the condensate amplitude involved in the coherence absorption process \( q_1^{eq} \) typically less than 0.5), we make use here of the standard weak-turbulence theory.

In the example of fig. 3b we considered \( M = 3 \) fields, in which the density of particles \( N_1 \) of \( \psi_1 \) was held fixed, while the densities \( N_{2,3} \) were varied from 0 to \( N_1 \) (with \( N_2 = N_3 \)). The initial energies per particle were assumed identical, \( U_j(t = 0) = 7 \) (i.e., \( \lambda_3,3(t = 0) \approx \lambda_1(t = 0) \)). The value \( U_1 = 7 \) corresponds to the critical energy \( U_{tr} \) for condensation of \( \psi_1 \) in the absence of \( \psi_{2,3} \). Accordingly, no condensation occurs for \( N_{2,3} = 0 \), while the presence of small-amplitude fields \( (N_{2,3}/N_1 \geq 0.3) \) leads to a significant fraction of condensed particles \( q_1^{eq} \approx 0.45 \) (see fig. 3b). Note that the condensate amplitude \( q_1^{eq} \) keeps a constant value if the system size is increased (keeping the particle density constant), a feature which is consistent with the fact that wave condensation occurs in the thermodynamic limit for \( D = 3 \) [5,9]. We emphasize that if the fields have the same amplitudes \( (N_{2,3} = N_1) \), the condensation of \( \psi_1 \) is suppressed \( (q_1^{eq} \approx 0) \), since in this case there are no small-amplitude fields to absorb the incoherence of \( \psi_1 \).

The corresponding values of the equilibrium entropies \( S_{1}^{eq} = \int \log(n_{1,j}^{eq}) dk \) of the fields \( \psi_j \) are shown in fig. 3c. Remarkably, the condensation of \( \psi_1 \) is characterized by a reduction of the entropy \( S_{1}^{eq} \). Actually, it is thermodynamically advantageous for the system to “organize” the field \( \psi_1 \), because this allows the small-amplitude fields...
\(\psi_{2,3}\) to reach a “more disordered” state. As a result, the reduction of \(S_1^eq\) is characterized by a large production of \(S_2^eq\), so that the total entropy \(S^eq = \sum S_i^eq\) results to be a concave increasing function of the extensive variable \(N = \sum N_i\) (fig. 3c), consistently with the fundamental properties of the entropy [13]. The temporal evolutions of the nonequilibrium entropies \(S_j(t)\) corroborate the same idea: an increase of entropy \(S(t)\) in the system requires the formation of a coherent plane wave (condensate) in the high-amplitude field \(\psi_j\), a feature characterized by a reduction of \(S_j(t)\) (see fig. 2c).

Considering the fact that finite-temperature Bose gases may be described classically in terms of incoherent fields satisfying eqs. (1) [15], the phenomenon of coherence absorption is expected to be also relevant for a genuine quantum mixture of thermal Bose gases. Note in this respect that the theorem of energy equipartition does not apply to a system obeying quantum statistics [13]. Contrary to a classical gas system, a quantum Bose gas may thus exhibit the coherence absorption process without violating the second law of thermodynamics. Let us consider a system composed of an uncondensed species \(\psi_1\) at its critical temperature \(T = T_c\), and suppose that a small fraction of uncondensed particles \(N_{j\not=1}\) are added in the system with an energy per particle \(U_j = U_1\). As a result of the coherence absorption process, the dilute species \(\psi_{j\not=1}\) are expected to lead to a condensation of the dense species \(\psi_1\). We verified this conjecture by considering the equilibrium properties of a mixture of ideal and uniform 3D Bose gases [13], \(N_j/N_1 = T_3^j/2g_{3/2}(z_j)/(g_{3/2}(1)r_j^{3/2})\),

\[
\frac{\hat{E}^eq}{N_1} = \frac{3T_3^{5/2}}{2g_{3/2}(1)} \left[ g_{3/2}(1) + \sum_{j\not=1} \frac{g_{3/2}(z_j)}{r_j^{3/2}} \right],
\]  

(9)

where \(\hat{E}_1^eq\) is the fraction of condensed particles in the dense species. The function \(g_p(z) = \sum_{z=1}^\infty z^p/\Gamma(p)\) refers to the Bose function [1], \(z_{j\not=1}\) being the fugacity, \(\hat{E}^eq = E^eq/k_BT\), \(T = T/T_c\) and \(r_j = m_1/m_j\). Following the same procedure outlined above through fig. 3b, these equations are solved for different values of \(N_j/N_1\), holding \(N_1\) constant. As remarkably shown in fig. 3d, the condensation of \(\psi_1\) is solely induced by the presence of \(\psi_{1\not=1}\). This shows the relevance of the coherence absorption effect for a system of quantum Bose gases. Note that \(\hat{E}_1^eq\) may be significantly increased if \(m_{j\not=1} > m_1\) (e.g., \(\hat{E}_1^eq \sim 0.35\) for \(r_j = 0.5\)).

In summary, we reported a previously unrecognized process of coherence absorption, in which small-amplitude fields are shown to absorb the incoherence of the system, thus leading to the condensation of the high-amplitude field. Given the universal character of the vector NLS equation in nonlinear systems, the reported phenomenon has a natural importance in various branches of physics [2,10,11]. In the context of thermal Bose gases, we showed that the coherence absorption process may be exploited to achieve the condensation of a dense species owing to its nonlinear interaction with a dilute species.

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REFERENCES